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A PHENOMENON STABILIZING A PLANE FLAME FRONT

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The conditions of stabilization of a plane flame front in the Darrie–Landau model on introduction of thermal resistance into it have been determined.

Keywords: *plane flame front, absolute instability, negative thermal resistance, isobaric heat supply, vibrational system, stabilization conditions, linear theory, perturbation.*

Introduction. L. D. Landau in his work [1] and independently Darrie [2] have solved the problem of the hydrodynamic stability of a plane flame front that turned out to be absolutely unstable. However, contrary to the conclusions of the theory [1] considered in a number of monographs, e.g., [1–5], the flame is rather often stable [2], i.e., stable steady-state fronts of a laminar flame are observed in experiments.

G. H. Markstein, by having introduced the dependence of the rate of combustion on the flame front curvature [3], has managed to find the flame stability for small wavelengths. The action of acceleration on the flame front may exert both a stabilizing and a destabilizing [2–4] influence. In Markstein's experiments with a pipe placed vertically [4] the flame stability was favored by the gravity acceleration g directed against the flow motion.

B. V. Raushenbakh [4] was the first to note that the flame front instability according to L. D. Landau's work [1] and the associated cellular structure of this front [3] do not directly relate to vibration combustion, i.e., to the excitation of thermoacoustic vibrations in pipes. This has necessitated the construction of quite a different solution of the problem of vibration combustion. With this goal in mind, in [6] a pressure characteristic of heat supply $H(Q)$ was introduced that allowed one to determine that part of the supplied heat which is converted into the head of the flow at different flow rates. Among the multitude of reasons favoring the formation of the ascending branch of the $H(Q)$ characteristic, the presence of which gives rise to the excitation of self-oscillations of vibration combustion [6], is the appearance of the descending branch of the losses of the head $h_{\text{th}}(Q)$ caused by the thermal resistance originating because of the heat supply on fuel combustion.

Thus, there is the equivalence of the needed conditions $\frac{dh_{\text{th}}}{dQ} < 0 \Leftrightarrow \frac{dH}{dQ} > 0$ being the reason for the excitation of vibration combustion.

Below, we will consider sufficient conditions of a stabilizing influence of a plane flame front on an increase in the thermal resistance $h_{\text{th}}(Q)$ [7] caused by heat supply on the surface of gasdynamic discontinuity [2] that divides the flow into a cold and heated portions.

Equations of L. D. Landau's Problem and Its Conditions. Unsteady-state motions before and after the flame front are described by the system of equations

$$\frac{\partial u'}{\partial t} + u^{\text{st}} \frac{\partial u'}{\partial x} = -\frac{1}{\rho^{\text{st}}} \frac{\partial P'}{\partial x}, \quad \frac{\partial v'}{\partial t} + v^{\text{st}} \frac{\partial v'}{\partial y} = -\frac{1}{\rho^{\text{st}}} \frac{\partial P'}{\partial y}, \quad \frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} = 0 . \tag{1}$$

L. D. Landau obtained the solution for the problem of the stability of a plane front represented by the surface of gasdynamic discontinuity with the following conditions:

$$u_1' = \frac{\partial x_{\text{f}}}{\partial t}, \quad u_2' = \frac{\partial x_{\text{f}}}{\partial t}, \tag{2}$$

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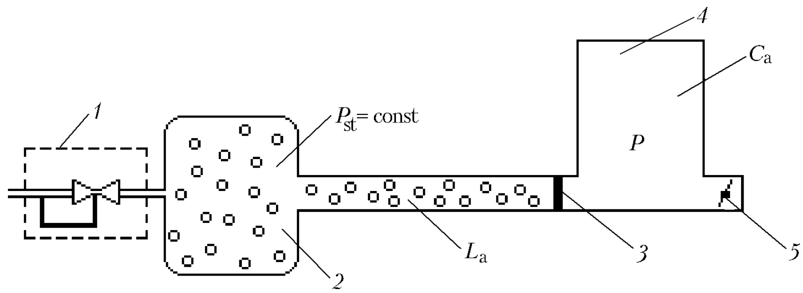

Fig. 1. Schematic diagram of the vibrational loop of a discrete system: 1) pressure regulator; 2) vessel filled with a mixture of gases; 3) plane flame front; 4) vessel of the force line; 5) resistance of the connected network.

where $x_f(y, t) = d \exp(\omega t + iky)$ is the shape of the perturbed flame front; with the condition of the equality of shear velocities defined by the equation

$$v_1' + u_1^{st} \frac{\partial x_f}{\partial y} = v_2' + u_2^{st} \frac{\partial x_f}{\partial y} ; \tag{3}$$

and with the equality of the deviations of pressure on both sides of the flame front

$$P_1' = P_2' , \tag{4}$$

which in our problem will be altered by taking into account the losses $h_{th}(Q)$ caused by the thermal resistance [7] of a plane flame front, whereas conditions (2)–(3) remain unchanged.

Specific Feature of the Behavior of a Vibrational System on Deviation of Its Parameters from Stationary Ones. To illustrate the behavior of a discrete dynamic system we will consider the simplest vibrational contour (Fig. 1), into which a mixture of gases is supplied under pressure P_{st} to undergo combustion in a plane flame located in front of the entrance to a lumped capacitance. The pressure characteristic of the vibrational contour (Fig. 1) is defined as $H(Q) = P_{st} - R(Q) - h_{th}(Q)$. The ascending branch of the function $H(Q)$ is formed because of the descending branch of the resistance $h_{th}(Q)$.

The system of equations that determines the motion in the pneumosystem considered coincides formally with the equations of the surging theory [8]:

$$L_a \frac{dQ}{dt} = H(Q) - P , \quad C_a \frac{dP}{dt} = Q - \varphi(P) , \tag{5}$$

whereas the inversion of the function $\varphi(P)$ [8] determines the characteristic of the network $h_n(Q)$ connected to the vibrational contour.

On deviation of the flow parameters from the values corresponding to a stationary regime in the region of the ascending branch of $H(Q)$, in the system either self-oscillations or a steady-state regime set in. The process of its development is vibrational, with a decrease in the flow rate to its steady-state value (Fig. 2). This is accompanied by an increase in the frequency of vibrations in the course of their damping [9]. Therefore the dependence of the thermal resistance $h_{th}(Q)$ in the process of the development of a stationary regime also appears to be vibrational with the frequency of the change in the flow rate. Solutions of the equations of the system with the distributed parameters given in [2–4] are also vibrational with a frequency ω.

Statement of the Problem and the Result of Its Solution. We consider the conditions of stabilization of the flame plane front in the Darrie–Landau model upon introducing, into it the thermal resistance [7], which remained unknown also after the publication of the above-mentioned fundamental initial model in [1]. As is already mentioned above, proceeding from the foregoing, in the present work the conditions (2) and (3) of L. D. Landau's problem are preserved, but into condition (4) we introduce the thermal resistance of a plane flame $P_1^* - P_2^* = h_{th}(u_1)$, from which we determine the difference of static pressures:

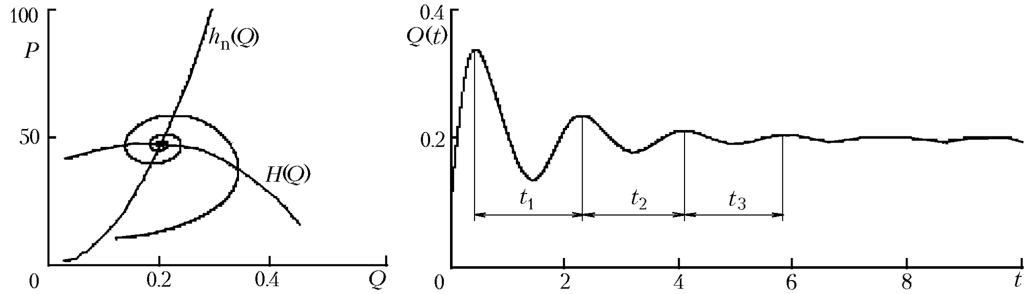

Fig. 2. Integral curves of the stable focus and the character of change in the flow rate $Q(t)$ on development of a stationary regime, where $t_1 > t_2 > t_3$ is the decrease in the period of decaying vibrations. P, Pa; $Q(t)$, m^3/sec.

$$P_1 - P_2 = h_{th}(u_1) + \frac{\rho_2 u_2^2}{2} - \frac{\rho_1 u_1^2}{2} . \tag{6}$$

Since heat supply takes place in the plane of the discontinuity surface, it [4] should be considered as isobaric, for which, according to [10], $h_{th}(u_1) = \frac{\rho_1 u_1^2}{2}\left[1 - \frac{T_2}{T_1}\right]$. Therefore Eq. (6) can be written in the form

$$P_1 - P_2 = (k_{th} - 1)\frac{\rho_2 u_2^2}{2} - \frac{\rho_1 u_1^2}{2} , \tag{7}$$

where $k_{th} = 1 - \frac{T_2}{T_1}$.

In the problem on the stability of the plane flame front the conditions $\rho_j = \text{const}$ ($j = \overline{1, 2}$) are adopted for flows before and after the flame front [2–4]. In this connection, taking into account this condition, we linearize Eq. (7) and write it in the form

$$P_1' - P_2' = (k_{th} - 1)\,\rho_1 u_1^{st} u_1' + \rho_2 u_2^{st} u_2' . \tag{8}$$

We introduce the designation $k_{th} = \left(k_{th}^{st} + k_{th}^{st}\omega\right)\rho_1 u_{th}^{st}$ and also assume that $\chi_1 = \rho_1 u_1^{st}$ and $\chi_2 = \rho_2 u_2^{st}$. Then the dependence of the thermal resistance can be written as $h_{th}(u_1') = k_{th}\chi_1 u_1'$. Substituting the solution of the system of equations (1), which are given in [2], into conditions (2), (3), and (8), we obtain a system of four linear algebraic equations for a_1, a_2, b, and d:

$$\frac{k}{\rho_1(\omega + u_1^{st}k)} a_1 + \omega d = 0 , \quad \frac{k}{\rho_2(\omega - u_2^{st}k)} a_2 + b - \omega d = 0 ,$$

$$\frac{-k}{\rho_1(\omega + u_1^{st}k)} a_1 + \frac{k}{\rho_2(\omega - u_2^{st}k)} a_2 + \frac{\omega}{k u_2^{st}} b + k\left(u_1^{st} - u_2^{st}\right) d = 0 ,$$

$$\left\{1 + (k_{th} - 1)\,\chi_1 \frac{k}{\rho_1(\omega + u_1^{st}k)}\right\} a_1 - \left\{1 + \chi_2 \frac{k}{\rho_2(\omega - u_2^{st}k)}\right\} a_2 - \chi_2 b = 0 .$$

For the existence of nontrivial solutions of this system of equations, its determinant should be equated to zero, i.e.,

$$\begin{vmatrix} \dfrac{k}{\rho_1(\omega+u_1^{st}k)} & 0 & 0 & \omega \\ 0 & \dfrac{k}{\rho_2(\omega-u_2^{st}k)} & 1 & -\omega \\ \dfrac{-k}{\rho_1(\omega+u_1^{st}k)} & \dfrac{k}{\rho_2(\omega-u_2^{st}k)} & \dfrac{\omega}{ku_2^{st}} & k\left(u_1^{st}-u_2^{st}\right) \\ 1+(k_{th}-1)\,\chi_1\dfrac{k}{\rho_1(\omega+u_1^{st}k)} & -\left(1+\chi_2\dfrac{k\chi_2}{\rho_2(\omega-u_2^{st}k)}\right) & -\chi_2 & 0 \end{vmatrix} = 0\,. \qquad (9)$$

Since $T_2 = T_1 + \dfrac{q}{c_p}$, the quantity $k_{th} = 1 - \dfrac{T_2}{T_1}$ can be represented as $k_{th} = \dfrac{-qu_1(\omega)}{c_pT_1}$ or by a Taylor expansion

$$k_{th} = \frac{-qu_1(\omega)}{c_pT_1} = \left.\frac{-qu_1(\omega)}{c_pT_1}\right|_{\omega=\omega_{st}} - \left.\frac{1}{c_pT_1}\frac{dq}{du_1}\frac{du_1}{d\omega}\right|_{\omega=\omega_{st}}(\omega-\omega_{st}) + O(\omega^2)\,.$$

Thus, when we neglect terms of the order of $O(\omega^2)$, we obtain the following terms; k_{th}: $k_{th}^{st} = \dfrac{-qu_1(\omega_{st})}{c_pT_1}$ and $k_{th}^1 = \left.\dfrac{-1}{c_pT_1}\dfrac{dq}{du_1}\dfrac{du_1}{d\omega}\right|_{\omega=\omega_{st}}$, with $k_{th}^{st} < 0$ and the value of k_{th}^1 being dependent on the character of the functions $q(u_1)$ and $u_1(\omega)$, where the derivatives are calculated at the values of parameters corresponding to a stationary regime.

We will introduce the notation $\tilde{k}_{th}^{st} = \dfrac{\chi_2 - \chi_1(1-k_{th}^{st})}{\rho_1 u_1^{st}}$ and $\tilde{k}_{th}^1 = \dfrac{k\chi_1 k_{th}^1}{\rho_2}$. Then, expanding the determinant (9), we obtain an equation to determine the increment of the problem $\tilde{\omega}$:

$$\tilde{\omega}^2 + \left(\frac{2+\tilde{k}_{th}^{st}}{\alpha+1-\tilde{k}_{th}^1}\right)\tilde{\omega} + \frac{1-\alpha}{\alpha\left(\alpha+1-\tilde{k}_{th}^1\right)} = 0\,, \qquad (10)$$

where $\tilde{\omega} = \dfrac{\omega}{\alpha k u_1^{st}}$; $\alpha = \dfrac{\rho_1}{\rho_2}$ [2].

According to [11], the condition Re $\tilde{\omega} < 0$, which ensures the stability of a plane flame front, consists in the positiveness of the coefficients of Eq. (10) that are reduced to the fulfillment of the following inequalities:

$$\tilde{k}_{th}^{st} > -2\,, \quad \tilde{k}_{th}^1 < 1+\alpha\,. \qquad (11)$$

Allowing for the fact that according to the continuity equation $\chi_1 = \chi_2$, at a constant cross-sectional area of the channel the first inequality in (11) will be reduced to $k_{th}^1 < -2$ which with allowance for $k_{th}^1 = 1 - \left.\dfrac{T_2}{T_1}\right|_{\omega=\omega_{st}}$ is transformed into the inequality $T_2/T_1 < 3$.

Thus, the value of the temperature T_2 at which the first condition of the stability of the plane flame front holds depends on the temperature T_1, and under the conditions adopted in the initial problem [1] it is limited. The second condition of the stability of inequalities (11) is as follows: $k_{th}^1 < \dfrac{\rho_2(1+\alpha)}{\chi_1 k}$. Since $k_{th}^1 = \dfrac{-1}{c_pT_1}\dfrac{dq}{du_1}\dfrac{du_1}{d\omega}$, then

$$-\frac{1}{c_p}\frac{dq}{du_1}\frac{du_1}{d\omega}<\frac{\rho_2(1+\alpha)}{\chi_1 k}\,. \qquad (12)$$

On flow stabilization the derivative $\frac{du_1}{d\omega}<0$, since with a decrease in the velocity u_1 the frequency ω increases, as follows from Fig. 2, where the time of change in the damping velocity pulses decreases. Therefore under the condition of the decrease of the function $q(u_1)$, when the derivative $\frac{dq}{du_1}<0$, the second condition of stability (inequality (12)) holds without any limitations. Since with an increase in the velocity u_1, the quantity of the heat supplied decreases, this also causes a decrease in the temperature T_2, and the losses of the head $h_{\text{th}}(Q)$ increase. The stabilization of the flame front proceeds in this case similarly to the action of acceleration directed opposite to the flow [2–4]. However, in our case the stabilization of the flame front is favored by the thermal resistance $h_{\text{th}}(Q)$, the coefficient k_{th} of which increases with the flow rate Q.

Conclusions. The introduction of the thermal resistance on the surface of the gasdynamic discontinuity that depends on the change in the heat supply with the flow velocity u_1, as well as on the change in the value of velocity u_1 in the case of regime deviation from a stationary one, exerts, in a certain way, an influence on the flame front stability. The justification of the stabilization of a plain flame front is given and the conditions of its stability in the classified Darrie–Landau model because of the thermal resistance caused by the heat of combustion of a gas mixture are obtained.

NOTATION

a, b, d, arbitrary constants; C_{a}, acoustic compliance, m^3/Pa; c_p, specific isobaric heat capacity, J/(kg·K); g, gravity acceleration, m/sec^2; H, flow head, Pa; $H(Q)$, force characteristics, Pa; $h_{\text{n}}(Q)$, characteristic of the network connected to the vibrational loop, Pa; $h_{\text{th}}(Q)$, losses of head because of heat supply, Pa; $i=\sqrt{-1}$, imaginary unit; k, wave number of perturbation along the axis y, 1/m; k_{th}, proportionality factor in the function $h_{\text{th}}(u)$; k_{th}^1, coefficient at the linear term in the Taylor expansion of $k_{\text{th}}(\omega)$ in the powers of ω; L_{a}, acoustic mass, Pa·sec^2/m^3; P, pressure, Pa; Q, volumetric flow rate of the flow up to the flame front, m^3/sec; q, supplied heat, J/kg; $R(Q)$, hydraulic losses in the vibrational loop, Pa; T, absolute temperature, K; t, time, sec; u, normal velocity component, m/sec; v, tangential velocity component, m/sec; x, longitudinal coordinate, m; y, coordinate along the normal to the flow, m; α, ratio of densities before the flame front and after it; ρ, density, kg/m^3; $\varphi(P)$, conversion of the function that determines the characteristic of the network, m^3/sec; χ, linearized value of velocity head, N·sec/m^3; ω, increment of the problem, 1/sec; Re $\tilde{\omega}$, real part of the complex number $\tilde{\omega}$. Subscripts and superscripts: a, acoustic value; f, flame front surface; n, network connected to the vibrational loop; st, stationary value of a parameter; th, thermal resistance; $'$, deviation of a value from a stationary one; 1, 2, parameters before the flame front and behind it; *, retarded flow; $\tilde{}$, relative value of decay increment.

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